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Vol. 57, No. 66, BULLETIN MASSACHUSETTS INSTITUTE OF TECHNOLOGY, May, 1922
Entered December 3, 1904, at the Post-Office, Boston, Mass., as second-class matter, under act of Congress of July 16, 1894

Pub. Serial No. 87

PUBLICATIONS

OF THE

Massachusetts Institute of Technology

Contribution from
Division of Industrial Co-operation and Research

Serial No. 1 May, 1922

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REPRINTED FROM

THE TECH ENGINEERING NEWS

ISSUE OF JUNE, 1922

PHOTO-ELASTICITY AND ITS APPLICATION TO ENGINEERING PROBLEMS

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One of the important and often difficult problems with which an engineer has to deal is the determination of the shape and the dimensions which

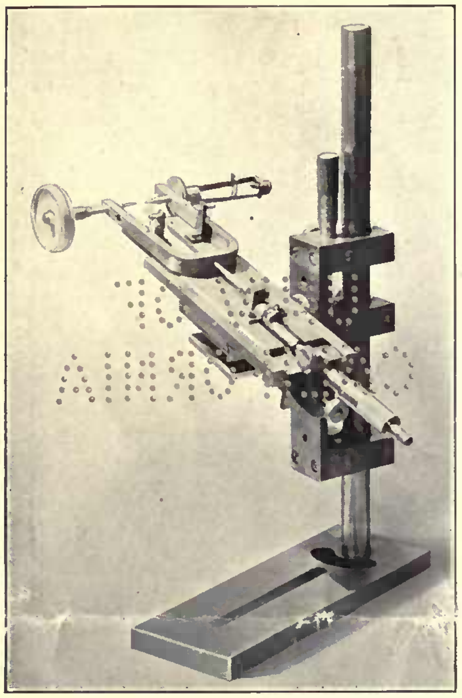


FIG. 1
DR. COKER'S LATERAL EXTENSOMETER

he must give to the different parts of a proposed construction. Several factors have to be considered, but the dominant ones are safety and economy.

As we know, materials yield at points where the stress exceeds the elastic limit. If we were able to make a complete determination of what the stress would be at each point of the structure, it would be easy to give to all parts adequate dimensions; a minimum of material would be wasted and safety would be secured.

Notwithstanding the progress made in mathematical treatment of elastic problems, the determination of the stress distribution remains in most cases an unsolved problem. Where theoretical solutions exist, even in the relatively simple cases, they usually lead to lengthy and intricate calculations, and it cannot be expected that even a well-trained engineer will use them in practice.

The uncertainty of knowledge of the stress distributions, on which the determination of the dimensions of the different parts of a construction has to be based, requires the use of an excessive factor of safety. All the material added above that strictly necessary means waste. Accidents frequently occur which show that, notwithstanding all precautions, safety in design has not been secured.

The object of the mathematical theory of elasticity is the analytical determination of this stress distribution. Photo-Elasticity, consisting of recently developed methods of optical investigation of the stress distribution, leads to an experimental solution for all two-dimensional elastic problems, provided the material used is isotropic and obeys Hooke's law of linear proportionality between stress and strain, *i.e.* within the elastic limit of the material.

As we know, the number of cases, for which a complete mathematical solution for the determination of stress

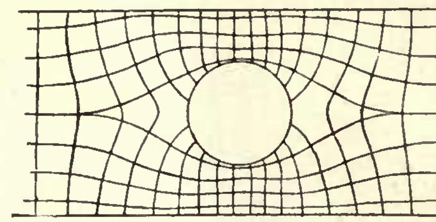


FIG. 4
LINES OF PRINCIPAL STRESS

distribution exists, is limited, and the calculations worked out in practice for the computation of stability are, in general, an adaptation of the ideal or incomplete theoretical solutions to the needs of engineering practice. The simplifying assumptions, which have to be made, may be seen, by closer analysis and especially by photo-elastic investigation, to be quite often of doubtful accuracy, and may lead, especially in new or unusual problems, to dangerous approximations.

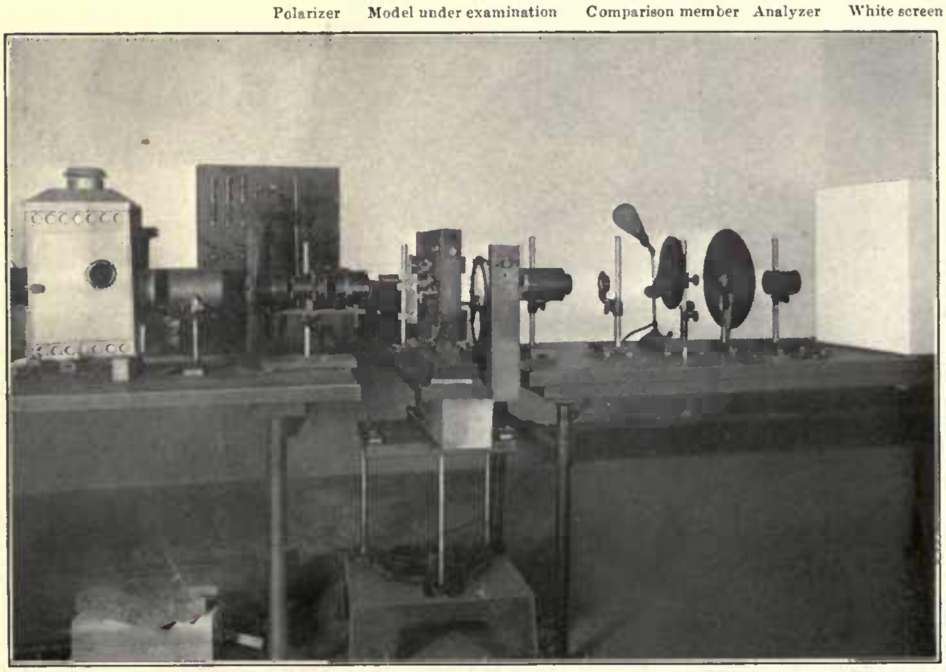
I. GENERAL PRINCIPLES OF ELASTICITY AND PHOTO-ELASTICITY

The state of stress at any point of a body is determined when the traction across every plane through the point is known. There exist at any point three orthogonal planes, across which the traction is purely normal and which are called the planes of principal stress. The normal tractions across those planes are called the principal stresses. So *the state of stress at any point is completely determined by the direction and the magnitude of the principal stresses* at the point under examination. These are therefore the elements which we determine in a stress analysis.

Most of the isotropic transparent bodies, such as glass, or still better such as xylonite (1) are optically inactive in their normal state, but show double refraction when put under stress. *The photo-elastic methods of stress investigation*, whose develop-

(1) Xylonite is camphorated nitro-cellulose, of the same kind as celluloid.

Polarizer Model under examination Comparison member Analyzer White screen



GENERAL VIEW OF APPARATUS USED IN THE OPTICAL STRESS ANALYSIS

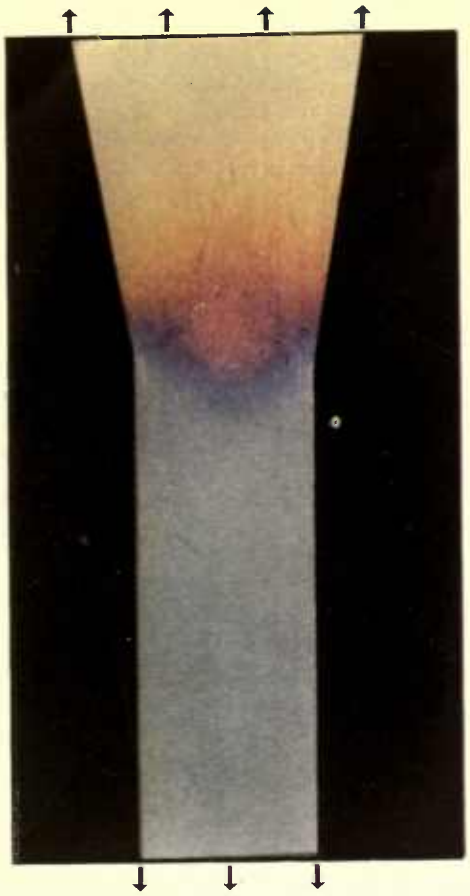


FIG. 2
BEAM OF RECTANGULAR CROSS SECTION UNDER LONGITUDINAL PULL

ment. as described in this article, has been mostly carried out under the direction of Prof. E. G. Coker (University College, University of London), *are based upon these temporary birefracting properties shown by transparent bodies when stressed.*

If plane polarized light is first passed through a stressed specimen of xylonite and afterwards through a second Nicol prism, whose principal section is parallel to the plane of polarization of the original beam of light, only the points where the principal stresses are respectively parallel and perpendicular to the principal sections of the crossed Nicols remain dark. This property enables us to determine *the directions of the principal stresses* at any given point.

If now we pass through the specimen circularly polarized light, by interference of the two component rays (which in the double-refracting specimen have suffered a relative retardation, depending at each point on the magnitude of the two principal stresses), a colored image is obtained.

By a comparison method, based upon the interposition in a suitable direction of a comparison member of constant cross section, put under uniform tension in an appropriate frame, we read on the dynamometer of the frame the value of *the difference of the principal stresses* at any given point.

Now, in the two dimensional elastic problems, the transverse deformation, *i.e.*, the deformation along a normal to the plane of the two principal stresses, is proportional to *the sum of those two stresses.* By means of Dr. Coker's lateral extensometer (Fig. 1) we measure this transverse deformation.

From the values of the differences and the sums of the principal stresses, we compute the separate values of each of them, so determining completely the elastic state.

The question naturally occurring to an engineer is whether the results, obtained on a transparent body such as xylonite, are of any value for engineering materials. It is shown by the general discussion of the equations of elastic equilibrium that *in the case either of plane strain or of plane stress,* in an isotropic body, obeying Hooke's law of linear proportionality between strain and stress, *the stress distribution is independent* of the moduli of elasticity and consequently *of the nature of the body.* So, the stress distribution experimentally determined on xylonite is the same in any other isotropic substance obeying Hooke's law, among others, in steel (2). Moreover these conclusions derived from the general theory of elasticity have been checked by experimental work.

II. PHOTO-ELASTIC STRESS ANALYSIS APPLIED TO:

A. *Some simple cases.*

Let us first consider the application of photo-elasticity to a few simple problems in order to make clear the different processes.

(2) Except, however, if the body is multiply-connected and the resultant applied forces do not vanish separately over each boundary. In this particular case the correction coefficients for passing from one isotropic substance to another, having different values for the elastic constants, may be experimentally determined. ("On Stresses in Multiply-Connected Plates," by L. N. G. Filon. British Assoc. Report, 1921.)

1. If we pull uniformly a xylonite *beam of rectangular cross section,* which is optically inactive in its normal state, the uniform color which will appear, as

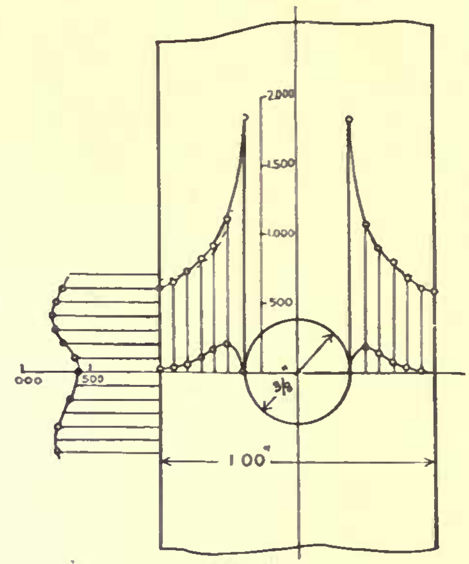


FIG. 5
THE PRINCIPAL STRESSES ACROSS THE MINIMUM CROSS SECTION AND ALONG THE OUTSIDE EDGE

it is put under stress, shows us (Fig. 2) where we have uniformly distributed tension.

If, in the central part of this same beam, previously under uniform tension, *a circular hole* is drilled, free from initial machining stresses, the image obtained (Fig. 3) shows that this internal discontinuity causes a very different distribution of stress. For one not familiar with these phenomena, the interpretation of such images may be somewhat difficult. It may be made easier by pointing out that the isochromatic lines or zones (lines or zones of same color) correspond to equal values of difference of principal

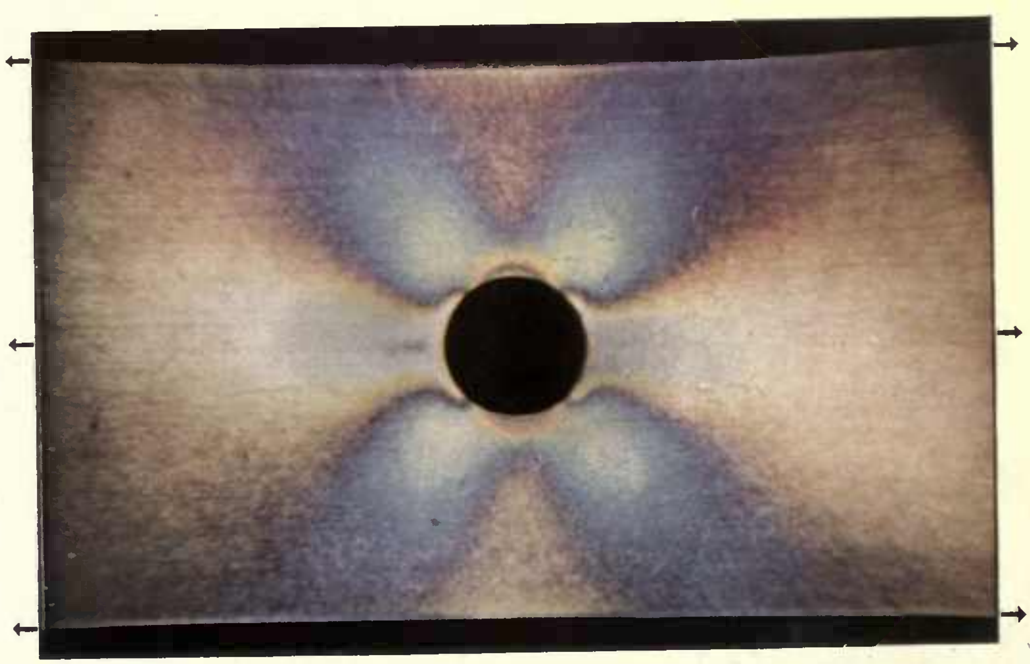


FIG. 3
A CIRCULAR HOLE IN A BEAM OF RECTANGULAR CROSS SECTION UNDER LONGITUDINAL PULL

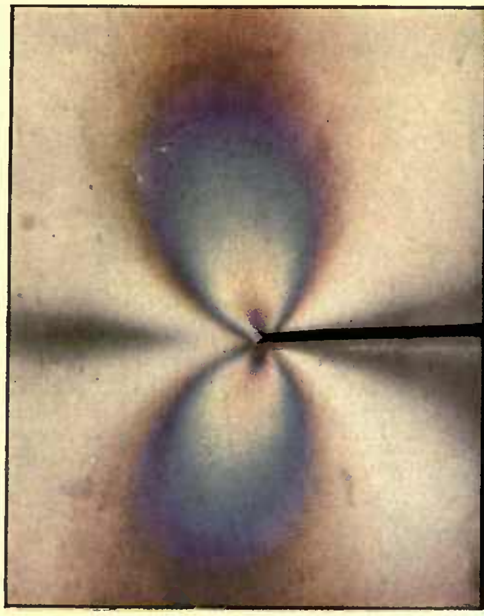


FIG. 8
Stresses at the Ends of a Crack

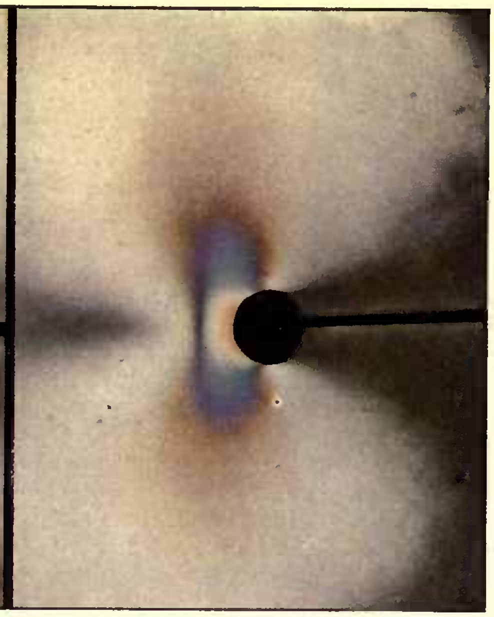


FIG. 9
A Circular Hole Drilled at the Ends of the Crack

A CRACK IN A BEAM OF RECTANGULAR CROSS SECTION UNDER LONGITUDINAL PULL

stresses. The image shows, among other things, that the stress is not uniform across the minimum cross section and that therefore the engineer makes an approximation, already in this very simple case, when he bases the calculation of the cross section on the mean stress through the active section.

The analysis of the stress distribution in a beam of rectangular section, in which a circular hole has been drilled (3), is shown on Figs. 4, 5 and 6.

Fig. 4 shows what are called the lines of principal stress. The tangent and the normal to those lines define at each of their points the directions of the principal stresses.

Fig. 5 shows the values of the principal stresses across the minimum cross section, and the polar diagram of Fig. 6 shows, with the boundary of the hole as origin for the values of the stresses, the value of the tangential stress at the boundary of the circular discontinuity.

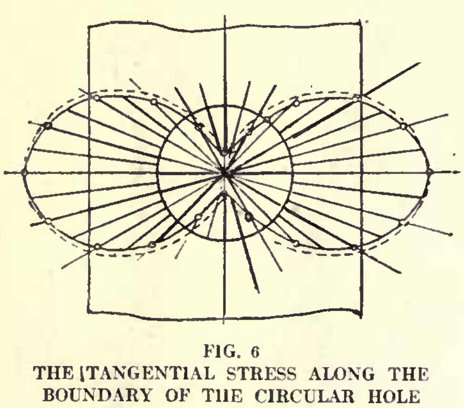


FIG. 6
THE TANGENTIAL STRESS ALONG THE BOUNDARY OF THE CIRCULAR HOLE

(3) Diagrams 4, 5 and 6 are taken from: "The effects of holes and semi-circular notches on the distribution of stress in tension members," by E. G. Coker (Proc. Physical Soc. London, vol. XXV, Part II).

What is most striking here is:

For the diagram of Fig. 5, that the maximum value of the stress, which occurs at the inside edge, is equal to about 1.8 times the mean stress. That ratio is the approximation made by the engineer when he calculates the section assuming that the stress is uniformly distributed.

For the diagram of Fig. 6, that the stress at the inside points of the hole where the longitudinal axis of the member cuts the boundary of the hole, the tangential stress is a compression.

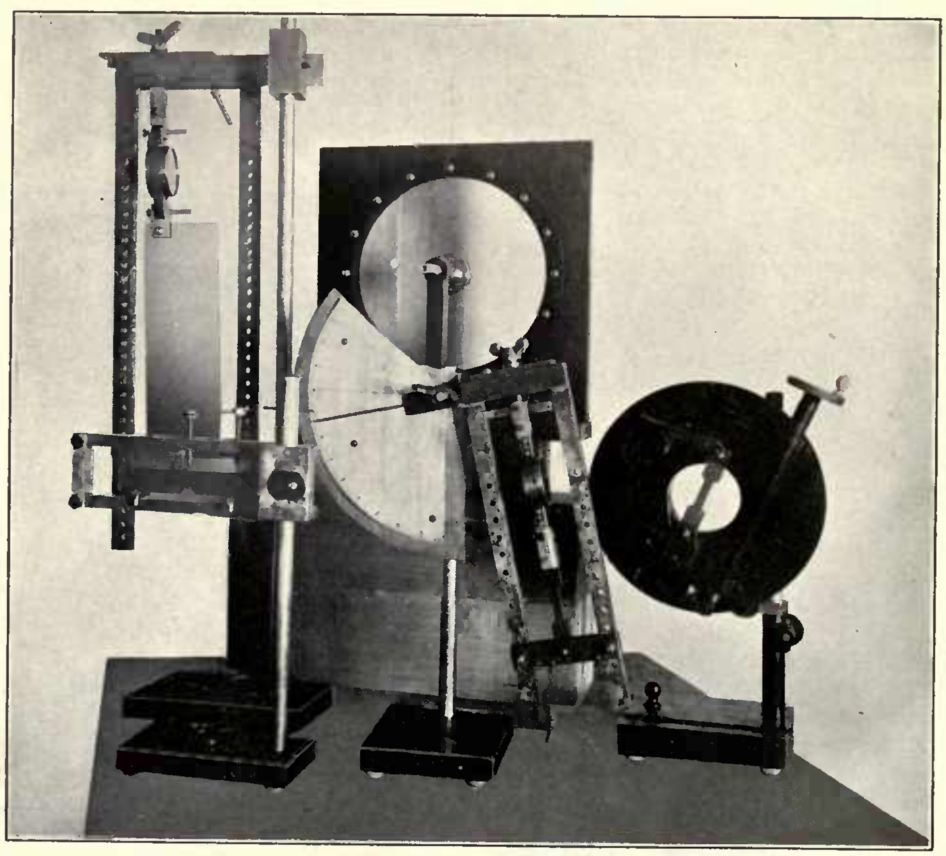
2. If in a similar beam of rectangular section, we cut *an elliptical hole*, whose major axis is perpendicular to the direction of the pull, the stress is, as in the previous case, concentrated near the boundary of the discontinuity; the maximum value of the stress occurs at the point of the boundary where the tangent to the ellipse is parallel to the direction of the pull.

The values of the stress at the boundary of the hole are given (4) by Fig. 7, (p. 84) where we see that the maximum stress is equal to four times the mean stress and that the compression at the end of the minor axis (90°) of the elliptical hole is equal to the mean stress.

The value of the maximum stress, at the point where the tangent to the ellipse is parallel to the direction of pull (0°), is, as has been mathematically shown (5), a function of the ratio of the lengths of the two axes. If we admit that a crack may be considered as an elliptical discontinuity such that one of the axes becomes very small, the fact that a crack usually keeps on extending, even if the member is only slightly stressed, is explained by this high stress concentration due to the

(4) Diagram 7 is taken from: "The effects of holes, cracks and other discontinuities in ships plating," by E. G. Coker and A. L. Kimball, Jr. (Trans. Inst. Naval Architects, London, 1920.)

(5) "Stresses in a plate due to the presence of cracks and sharp corners," by C. E. Inglis. (Trans. Inst. Naval Architects, London, 1913.)



FRAMES USED FOR STRESSING SMALL MODELS

high value of the ratio of the major axis to the minor. Fig. 8 shows the high concentration of stress occurring at the end of such a crack; and the way, well known to engineers, to limit the extension of a crack, which is starting, is to replace the ends of the elliptical discontinuity by a circular one, where the ratio of the axes is equal to one: by drilling two circular holes, we get an image as shown by Fig. 9, corresponding to a more uniform distribution of the stresses and consequently to smaller stress concentration. The replacement of the elliptical crack by two elliptical holes whose axes are normal to those of the crack has been shown to be still more advantageous.

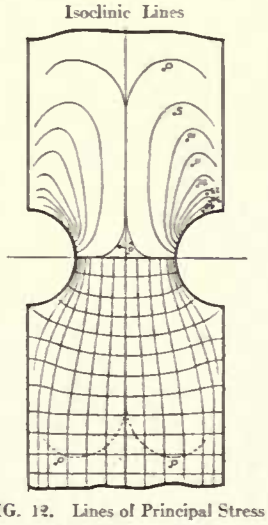


FIG. 12. Lines of Principal Stress

3. If again in a similar beam, we cut out at both sides two symmetrical *semi-circular notches*, we get the image shown by Fig. 10.

A V-notch (Fig. 11) shows a somewhat similar stress distribution. For the semi-circular notch, the lines of principal stress (6) are shown by Fig. 12. Fig. 13 (p. 84) shows the values of the principal stresses for a notch of ¼ inch radius across the minimum cross section, and Fig. 14 (p. 84) the values of the tangential stress along the semi-circular notch and along the outside parallel edge. Again here the value of the maximum principal stress is about 30% higher than the mean stress through the minimum cross section. Figs. 15 to 20 inc. (p. 85 and 96) give the values of the principal stresses (7) across the minimum cross section and along the edge of the lateral discontinuity, respectively for the V-notch, for the U-notch and for the Charpy notch (impact tests), in a beam of rectangular cross section under longitudinal pull.

(6) Diagrams 12, 13 and 14 are taken from: "Photo-elastic measurements of the stress distribution in tension members used in the testing of materials," by E. G. Coker. Proc. Inst. Civil Eng., London, 1921.)

(7) Diagrams 15 to 20 are taken from: "Stress concentrations due to notches and like discontinuities," by E. G. Coker and Paul Heymans, (British Assoc. Report, 1921) and "Etude par la photo-elasticimetrie de la distribution des surtensions dues a certaines discontinuites dans les pieces soumises a traction," by Paul Heymans. (Academie Royale de Belgique, Classe des Sciences, Brussels, 1921.)

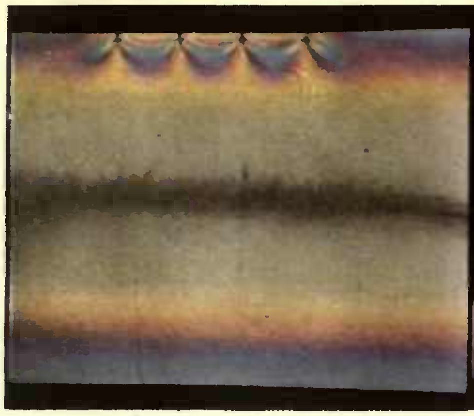


FIG. 21
FOUR PARALLEL SCRATCHES AT THE SURFACE OF A BENT BEAM

FIG. 25
A XYLONITE MODEL OF A GEAR WHEEL

The maximum stress depends on the radii of the curves at the bottom of the notch. If for the V-notch this radius becomes small, we have a scratch, as those shown by Fig. 21, which represents the effect of four parallel scratches at the surface of a bent beam. The higher maximum stress which is revealed by this photo-elastic analysis shows why a scratch, which is surely not an appreciable reduction of the active section, weakens considerably a member, especially if it is made of brittle material, such as glass or hardened steel, in which practically no redistribution of stress occurs between the elastic limit and the breaking point.

B. Some specific engineering problems

A great number of problems occurring in structural engineering, such as bridge construction, design of transmission towers, naval architecture, etc., either are or may be decomposed into two-dimensional elastic problems, and therefore, however hyperstatic and indeterminate they may be, are capable of being solved, as completely as desired, by the photo-elastic methods.

1. Fig. 22 (p. 96) for instance shows the model of *a simple truss*, which has been used for experimental purposes (Prof. Coker's laboratory) under different combinations of applied loads. The stress analysis at the different joints is of course the most interesting. The image obtained by photo-elastic analysis (8) shows immediately that the

(8) "La Photo-elasticimetrie, ses principes, ses methodes et ses applications," by Paul Heymans, (Bull. Soc. Belge Ing. et Ind., Brussels, 1921), p. 113.

FIG. 10
Lateral Semi-Circular Notch

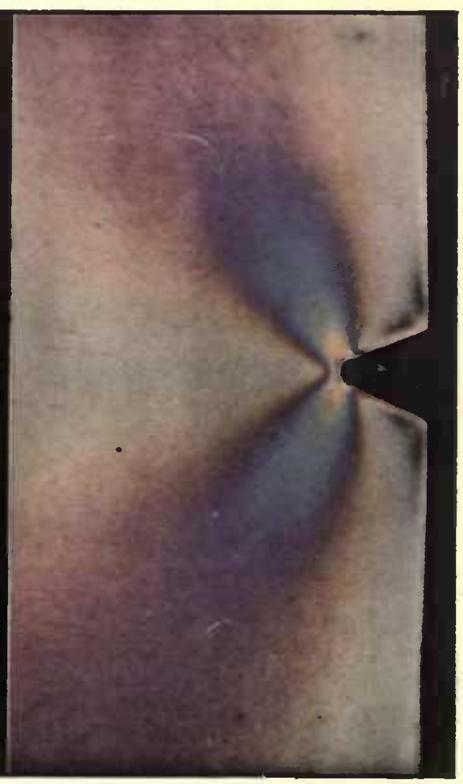


FIG. 11
Lateral V-Shaped Notch

LATERAL NOTCHES IN A BEAM OF RECTANGULAR CROSS SECTION UNDER LONGITUDINAL PULL

FIG. 26
A Xylonite Model under inside pressure due to the Shrinking on the Shaft

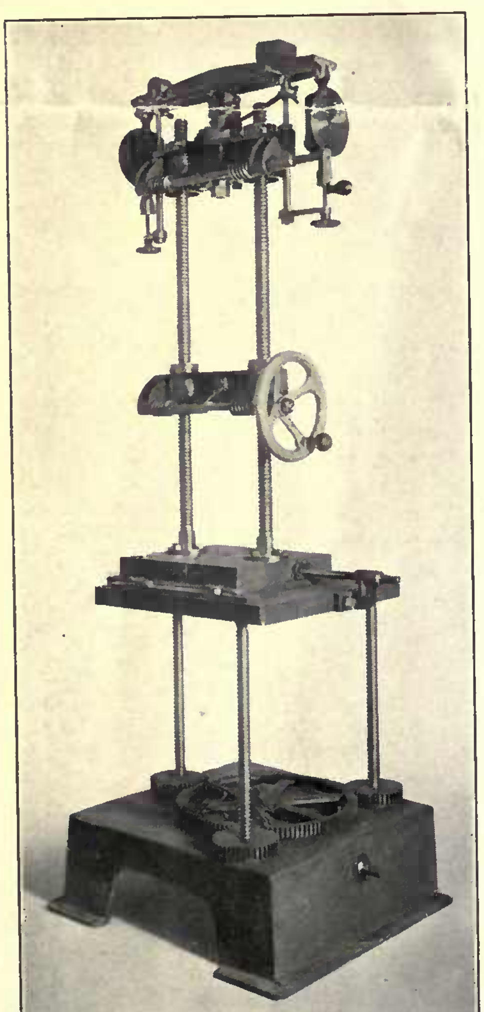
FIG. 27
The Xylonite Model when the Torque is applied

PHOTO-ELASTIC STUDY OF GEAR WHEELS AND PINIONS

members are not, as assumed in calculations, under uniform stress. In fact, the image shows the actual state of stress at each point, due as well to the so-called primary as to the secondary stresses.

2. Application of photo-elastic analysis has been made successfully to *bridge construction.*

An interesting case is the one made and published by M. Mesnager, Chief Engineer of the Department of Bridges and Roads in Paris (9).



SPECIAL PHOTO-ELASTIC TENSION AND COMPRESSION MACHINE DESIGNED BY A. L. KIMBALL, JR.

Fig. 23 (p.100) shows a general view of the bridge and Fig. 24 represents the models and special frames used.

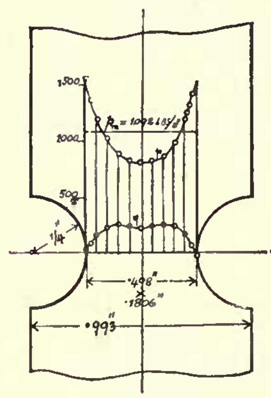


FIG. 13
THE PRINCIPAL STRESSES ACROSS THE MINIMUM CROSS SECTION

It is interesting to quote M. Mesnager (10), "We have used photo-elastic analysis in the laboratory of the Ecole des Ponts et Chaussées

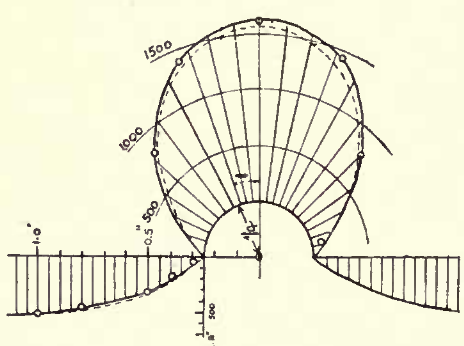


FIG. 14
THE TANGENTIAL STRESS ALONG THE BOUNDARY OF THE NOTCH AND ALONG THE OUTSIDE EDGE

(Paris) in order to check the calculations made for a bridge of ninety-five meters span, which was to be built over the Rhone at la Balme. . . . The amount of calculations made was considerable, and, while checking them, certain errors in the methods and in the figures had been detected. Complementary calculations, sent in as corrections of the first one, extended into a book of forty-eight pages. Were there no other errors which had been

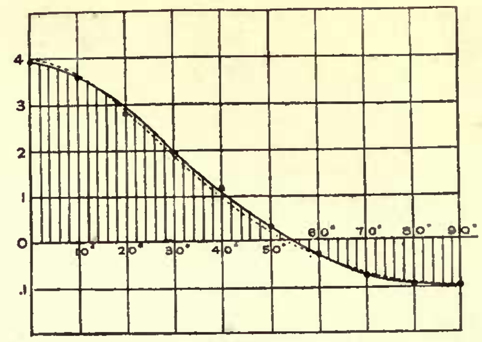


FIG. 7
THE TANGENTIAL STRESS ALONG THE BOUNDARY OF THE ELLIPTICAL HOLE

overlooked? Besides, for such an important construction in reinforced concrete, was it not advisable to make certain preliminary experimental verifications? Tests made in the laboratory on small metal models had given some interesting results, but most of them were incapable of interpretation.

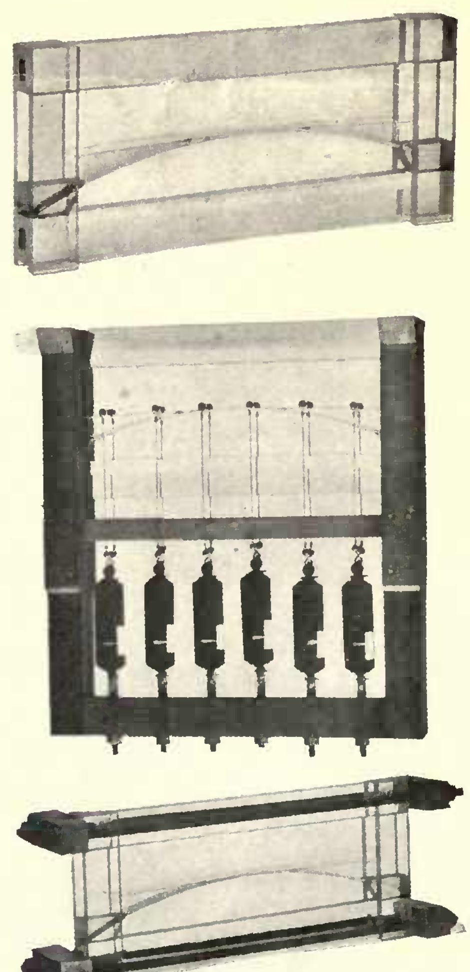


FIG. 24
THE GLASS MODEL AND THE FRAMES USED BY MESNAGER IN HIS PHOTO-ELASTIC ANALYSIS OF THE LA BALME BRIDGE

(9) "Utilisation de la double refraction accidentelle du verre a l'etude des efforts interieurs dans les solides," by M. Mesnager. (Annales des Ponts et Chaussees, Paris, 1913.)

(10) Translated from M. Mesnager, loc. cit., p. 178 and p. 135.

On the other hand, reliable experimental results were asked by the Acting Commission before decision. It is in those circumstances that, in order to come to a solution, I had a

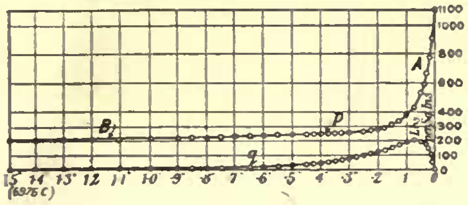


FIG. 15
THE PRINCIPAL STRESSES ACROSS THE MINIMUM CROSS SECTION (V-Notch)

glass model of the bridge made. This work, which cost less than one-thousandth of the price of the construction, finally enabled us to get the necessary verifications. . . . The

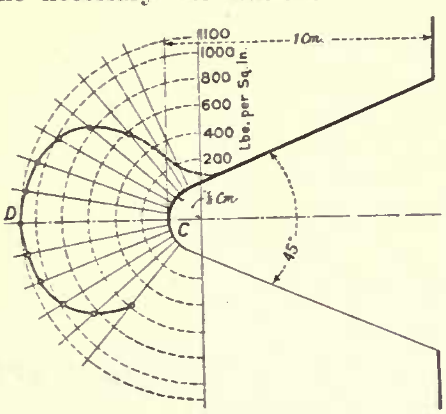


FIG. 16
THE TANGENTIAL STRESS ALONG THE BOUNDARY OF THE NOTCH

study of the reduced glass model enabled us to obtain accurate results in less than twenty days, whereas the calculus had required a time considerably longer without giving the same reliability."

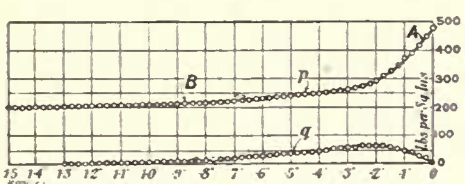


FIG 17
THE PRINCIPAL STRESSES ACROSS THE MINIMUM CROSS SECTION (U-Notch)

However at the time when this application of photo-elastic analysis was made, only the maximum stresses, which always occur at the edges where one of the principal stresses vanishes, could be obtained and, furthermore, Mesnager used glass, which gave him serious difficulties for the building of the models.

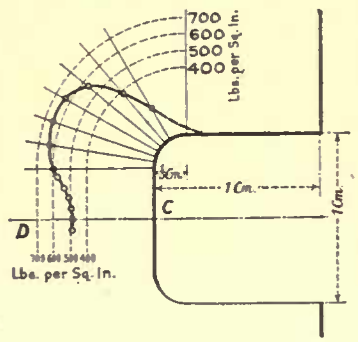


FIG. 18
THE TANGENTIAL STRESS ALONG THE BOUNDARY OF THE NOTCH

3. In the photo-elastic laboratory here at Technology an extended study of stress distribution in certain special *gear teeth and pinions* is now being carried out.

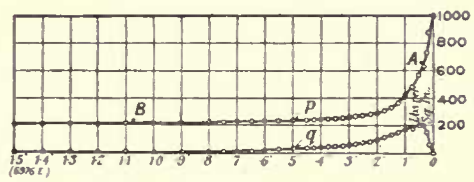


FIG. 19
THE PRINCIPAL STRESSES ACROSS THE MINIMUM CROSS SECTION (Charpy Notch)

Fig. 25 (p. 83) shows one of the xylonite models when no force is applied.

Fig. 26 (p. 84) gives the colored image obtained for the structure in position on the shaft.

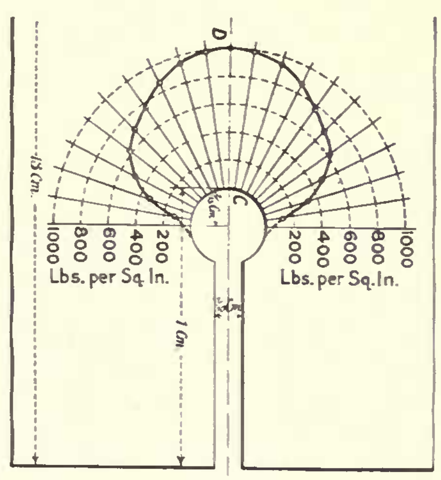


FIG. 20
THE TANGENTIAL STRESS ALONG THE BOUNDARY OF THE NOTCH

Fig. 27 (p. 84) gives the colored image when the torque is applied.

The work being still in progress, it would be premature to discuss results.

When one has examined the stress distribution in a certain number of members entering into a structure, he is impressed with the number and nature of the approximation employed in engineering problems. Empirical calculations of that kind, — and practically all contain simplifying assumptions, — may be relied upon only when they have been sufficiently checked by experience, — if, in other words, the factor of safety has been for each particular case adequately adjusted.

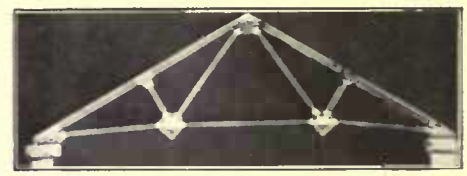


FIG. 22
A XYLONITE MODEL OF A SIMPLE TRUSS

For new types of construction, or where it becomes of importance to distribute the structural material in the most economical manner, these approximate calculations are often inadequate. It may be, as in airplane design, that the purpose is to lighten the structure, or as in the designs of steel and concrete constructions, to reduce the amount of idle material.

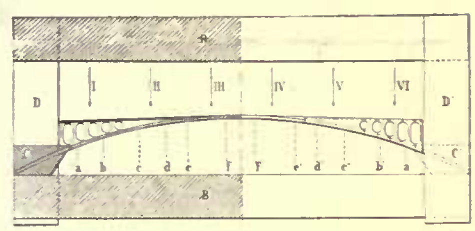


FIG. 23
GENERAL VIEW OF THE LA BALME BRIDGE

By the use of photo-elastic methods these objects can be obtained without compromising safety. The constantly recurring accidents caused by failures of mechanical devices and structures, emphasize the need of a method of analysis such as is now offered through the photo-elastic investigations.

The photo-elastic method of investigation has been developed principally by Dr. E. G. Coker of University College, University of London, with whom the writer has studied and to whom he is greatly indebted for inspiration and suggestions concerning this later work. The author also acknowledges the assistance of Mr. John T. Norton in taking many photographs including the autochrome color plates, used in this article, and of Mr. Carl Selig in cutting the xylonite models and the General Electric Company in loaning a portion of the apparatus.